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MAC-CPTM Situations Project

Angles of Incidence and Reflection

Prompt

During a geometry class for preservice middle school teachers, the following problem was posed.

> How big is the reflection of your face in a mirror? Is it the same size as your face, or is it larger or smaller, and if so, how much? … Use a ruler to measure the length of your face from the top of your forehead to your chin. Now hold a mirror *parallel* to your face and measure the length of your face's reflection in the mirror.… Hold the mirror closer to your face or farther away (but always *parallel* to your face), and repeat the measuring processes just described. The *position* of your reflection will probably change, but does the *size* of your reflection change or not?

From Beckmann, S. (2008). *Activities manual for Mathematics for elementary teachers (2nd ed.)*. Boston, MA: Pearson.

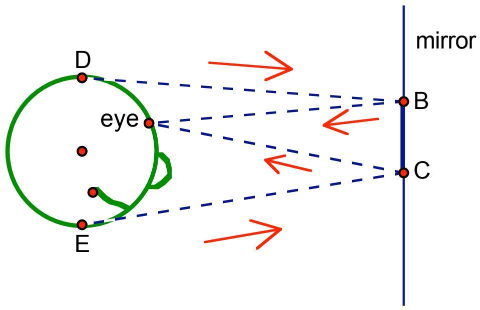
After working on this problem for a while, the preservice teachers realized that the reflection, as they measured it, was always ½ the length of their actual face. At the end of class, one of the students asked how this was possible. Don't objects that are further away from a mirror reflect smaller images than objects close to the mirror?

Mathematical Foci

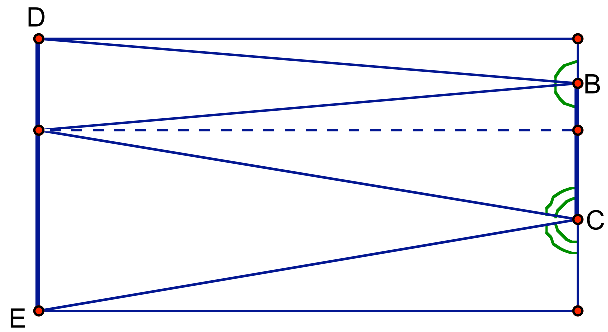
Mathematical Focus 1[1]

Using congruent triangles formed from the angles of light incidence and reflection, one can easily see that the reflection, as measured above, will always be ½ of the actual face length.

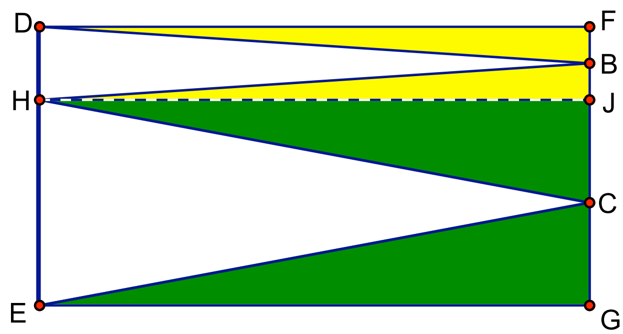
[1] Special thanks to Sarah Donaldson for sharing her write-up of this situation.



Regardless of how far the mirror is from the person's face, the length of BC will always be half the length of DE.



When light reflects, the angle of the light ray coming into the mirror is the same as the angle of the ray going out (i.e. the angle of incidence = angle of reflection). So the two marked angles at B are congruent and the two marked angles at C are congruent.



DFJH is a rectangle, so angles F and J are congruent (both 90 degrees), and segments DF and HJ are congruent. The same argument applies for rectangle HJGE. Therefore triangles DFB and HJB are congruent (yellow) and triangles HJC and EGC are congruent (green) by AAS. Therefore the measure of FB = the measure of BJ and the measure of JC = the measure of CG (corresponding parts of congruent triangles are congruent).

Because DFGE is a rectangle, segments DE and FG are congruent. Segment FG is comprised of segments FB, BJ, JC, and CG. Also, segment BC consists of BJ and JC. Together BJ and JC make up half the length of FG (see algebra below). Therefore the length of BC is half the length of DE.

$FB = BJ = x$
$JC = CG = y$
Since $DE = FG$ (opposite sides of a rectangle), $DE = 2x + 2y$.
$BC = x + y = (1/2)(2x + 2y) = (1/2)(DE)$
That is, BC is half the length of DE.

Mathematical Focus 2
Normal lines are used to determine angles of incidence and reflection. Perpendicular bisectors determine the normal lines needed to determine the location of the reflection in the mirror.

Mathematical Focus 3
Essential to this activity is that the person whose image was reflected in the mirror measure his or her own face, rather than having a classmate measure the face. However, even if another person did the measuring the result would be the same. (This focus would look at the reasons for this.)